DNN-003-026201 Seat No. ________

M. Phil. (CBCS) (Sem. II) Examination

April / May – 2015

Mathematics

Topology

Faculty Code : 003
Subject Code : 026201

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions: (1) There are five questions in this paper.
(2) Each question carries 14 marks.
(3) All questions are compulsory

Q.1 Fill in the blanks: (Each question carries two marks)

(i) Every Z- ideal is the intersection of ....ideals.

(ii) Every free ideal in C ($\mathbb{N}$) contains the ....function of every singleton subset of $\mathbb{N}$.

(iii) The set $I = \{f \in C(\mathbb{R}) \mid f(\frac{1}{2}) = 0\}$ is a . ... .ideal in $C(\mathbb{R})$.

(iv) If I is z – ideal that contains a prime ideal in C(X) then I is a.......... ideal.

(v) If I and J are fixed ideals in C(X) then $I \cap J$ is a ....ideal in C(X).

(vi) If M is a maximal ideal in C (X) then $M \cap C^*(X)$ is a ....ideal in $C^*(X)$.

(vii) If every maximal ideal in $C^*(X)$ is fixed then X must be a ....space.

Q.2 Attempt any two of the following:

(a) Prove that every ideal in C(X) is fixed if and only if every ideal in $C^*(X)$ is fixed. 7

(b) Prove that every fixed maximal ideal in C(X) is of the form $M_p = \{f \in C(X) : f(p) = 0\}$ form some $p \in X$. 7

(c) Prove that a free maximal ideal in $C^*(\mathbb{N})$ cannot be the intersection of an ideal in $C(\mathbb{N})$ with $C^*(\mathbb{N})$. 7

Q.3 All are compulsory:

(1) Prove that a C^*- embedded subset S of X is C – embedded in X if and only if it is completely separated from every zero set disjoint from it. 7

(2) Prove that any two sets A and B contained in disjoint zero sets of X are completely separated in X. 3

(3) Give an example of a Z – ideal in C ($\mathbb{R}$)which is contained in a unique maximal ideal of C ($\mathbb{R}$). 4

OR

Q.3 All are compulsory:

(1) Prove that the following statements are equivalent for the space $\mathbb{R}$ of real numbers and a subset C of $\mathbb{R}$. 6

(i) C is C – embedded in $\mathbb{R}$.

(ii) C is a zero set.

(2) Suppose X is a dense subspace of T. Prove that X is C^*- embedded in T iff $Cl_T Z_1 \cap Cl_T Z_2 = \emptyset$ for any two disjoint zero sets Z_1 and Z_2 of X. 5

(3) Prove that every ideal in C ($\mathbb{N}$) is a Z - ideal. 3

Q.4 Attempt any two of the following:

(1) Define $\bar{Z}$ for $Z \in Z$ (X). Prove the following: 7

(i) The family $\{\bar{Z}: Z \in Z\ (X)\}$is a base for closed sets for some topology on βX.

(ii) $Cl_{\beta X}\, Z = \bar{Z}$ for every $Z \in Z$ (X).

(iii) $\bar{Z} \cap X = Z$ for every $Z \in Z$ (X).

(2) Prove the following: 7

(i) X is dense in βX.

(ii) βX is compact and Hausdorff.

(3) Prove that 7

(i) X is C^*- embedded in βX.

(ii) X is disconnected if and only if βX is disconnected.

Q.5 Do as directed: (Each question carries two marks) 14

(i) Give an example of a subset of $\mathbb{R}$ which is a zero set in $\mathbb{R}$ (with lower limit topology) such that it is not a zero set in $\mathbb{R}$ (with standard topology)

(ii) State if the set of irrational number is a zero set or not.

(iii) Give an countable subset of $\mathbb{R}$ which is not a zero set.

(iv) Determine the smallest zero set in $\mathbb{R}$ *containing the set* $\mathbb{R} \setminus \mathbb{Q}$.

(v) Give an example of a free ideal in C^* ($\mathbb{N}$).

(vi) Give two continuous functions f and g defined on $\mathbb{R}$ such that Z (f) = Z (g) but f $\neq$ g.

(vii) Give the definition of a compactification of a space and give the characteristic property of the stone cech compactification of a space X.
